Journal of Interdisciplinary and Multidisciplinary Research

Journal of Interdisciplinary and Multidisciplinary Research (JIMRES) www.jimres.org

E-ISSN: 1936-6264 | P-ISSN: 1945-3019

Check for updates

Vertex Antimagic Edge Slither Labeling Of Multi Leaf Caterpillar Graphs

Abstract: Graphs are one of the most fascinating topics in the field of mathematics having a wide range of applications that stretch from extreme science and technology to daily life. Many graph theoretic concepts have proved to be effective when collaborated with other fields of science. One such notion in graph theory is the caterpillar graphs. The structure of the caterpillar graph is easy to construct and comprehend. Therefore, applying caterpillar graphs different conceptions in graph theory is intriguing. to One such intriguing concept is that of labeling. In this paper, the applicability of vertex antimagic edge slither labeling to the caterpillar graph is attempted.

R. Sreenivasan [1]

Affiliation

[1] Assistant Professor, PG And Research Department of Mathematics, Agurchand Manmull Jain College, Meenambakkam, Chennai, India.

Corresponding Author

R. Sreenivasan * - sreenivasan.raghavan@gmail.com

Article History:

Received Date : Mar 04, 2026
Revised Date : Apr 23, 2026
Accepted Date : May 30, 2026
Published Date : Jun 05, 2026

Keywords

Vertex Antimagic Edge Slither Labeling, Caterpillar graph

1.Introduction

The graph theoretic contents play a vital role in elucidating the complex situations into simple and suave nature. The idea of graphs has thrown light on many critical philosophies that human mind finds hard to fathom. These philosophies were once considered imaginary and with the advent of graphs, they are being represented in the form of a diagram with dots and lines. With much more in the offing, graphs have become the fulcrum to gain better understanding of intricate structures.

2. Vertex Antimagic Edge Slither Labeling

The perception of labeling in graph theory has been widely considered as the most effective form of representing various real life situations. The idea of labeling originated from the observation of magic labeling by graph theorist J. Sedlacek [8] who was the first to introduce this concept. His work and conclusions gave rise to numerous innovations of such as the vertex magic labeling formulated by J.A. Macdougall, M.Miller, Slamin and W.D.Wallis [5], antimagic labeling defined by N. Hartsfield and J. Ringel [3]. The notion of vertex antimagic edge labeling was due to R. Bodendiek and G. Walther [1]. The compilation and collation of all these types of labeling was done by Joseph A. Gallian [2] in the book titled "A dynamic survey of graph labeling". Martin Baca and Mirka Miller [4], were the first researchers to give the definition for vertex antimagic edge labeling in graphs. They defined the graph G to an (a, d) antimagic edge labelled graph if it is connected and for a positive integer a and a non-negative integer d, there exists a bijective mapping given by $f_1: E \rightarrow \{1,2,\ldots,|E(G)|\}$ such that the induced mapping $h_1: V(G) \rightarrow W$, where $W = \{a, a+d, a+2d,\ldots, a+|V-1|d\}$ is also a bijection. Slither labeling is an innovation of vertex antimagic edge labeling in which the labeling of the edges follow a slithering pattern.

Journal of Interdisciplinary and Multidisciplinary Research
E-ISSN: 1936-6264 | P-ISSN: 1945-3019
Journal of Interdisciplinary and Multidisciplinary Research (JIMRES) www.jimres.org
E-ISSN: 1936-6264 | P-ISSN: 1945-3019

Check for updates

3. Caterpillar Graph

The definition of caterpillar graphs considered in this paper was given by P. Packiavathi, S. Balamurugan, nd R. B. Gnanajothi[7], is a graph which is formed from a path on k vertices by adding x_i pendant vertices to the i^{th} vertex of the path P_k. A caterpillar graph is a tree having the property that the removal of the leaves and incident edges results in a path P_k which is called the spine of the caterpillar. A caterpillar is said to be complete if every vertex on the spine is adjacent to at least one leaf of the caterpillar. R. Sreenivasan [9] has already proved the admittance of vertex antimagic edge slither labeling to P_n^m, m = 2,3. In this paper, the caterpillar path having m leaves on a path on n vertices is denoted by $P_n^m, m = 4, 5\ and\ 6$. A typical caterpillar graph is given as follows;

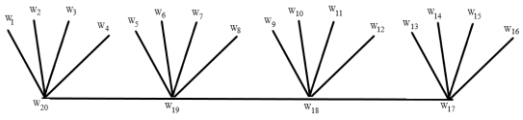


Fig 1. Caterpillar Graph P_4^4

4. Main Result

Theorem 1: The caterpillar graph P_n^m defined on a path on n vertices admits vertex antimagic edge slither labeling.

Proof:

CASE – 1: Consider the caterpillar graph P_n^m formed from a path of n vertices. There are m leaves that originate from each of the vertices of the path. Label the edges of the graph by slither pattern. Let the number of leaves in the graph be m = 4, 5. Consider the graph P_4^4. The slither labeling in P_5^2 is given as follows;

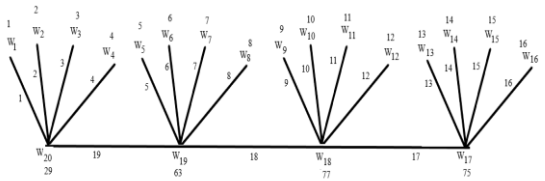


Fig 2. Caterpillar Graph P_4^4

The labeling of the edges of the graph are done with positive integers in a slither pattern. The label of a vertex is the sum of labels of the edges that are incident with the vertex. In the graph P_4^4 the vertex labels are all distinct and hence the graph admits vertex antimagic edge slither labeling.

CASE – 2: Consider the caterpillar graph P_4^5, the path on 4 vertices and five leaves originate from each vertex of the path. The slither labeling is given as follows;

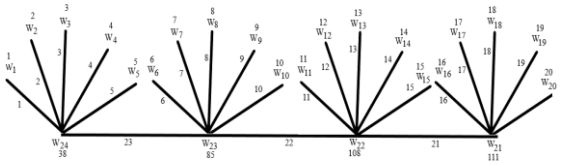


Fig 3. Caterpillar Graph P_4^5

In the given graph P_4^5, the labels of the vertices are all distinct and hence the graph admits vertex antimagic edge slither labeling.

From the cases discussed above, it has been proved that the graph P_n^m, m = 4,5 admits vertex antimagic edge slither labeling.

CASE – 3: Consider the caterpillar graph P_2^6, the path on 2 vertices and six leaves originate from each vertex of the path. The slither labeling is given as follows;

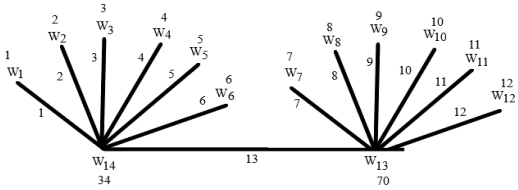


Fig 4. Caterpillar Graph P_2^6

5. Conclusion

In this paper, the caterpillar graph P_n^m with multiple leaves are taken and the applicability of vertex antimagic edge slither labeling has been proved for m = 4, 5 and 6. The same concept of slither labeling pattern can be tried for all caterpillar graph P_n^m m $\geq$ 7.

Conflict of Interest. There is none among the authors

Funding.
No external funding, private or government financial support or grant was received before, during and after this study.

References

[1] Bodendiek, R., & Walther, G. (1998). On arithmetic antimagic edge labelling of graphs. *Mitteilungen der Mathematischen Gesellschaft in Hamburg, 17*, 85–89.
[2] Gallian, J. A. (2018). A dynamic survey of graph labeling. *The Electronic Journal of Combinatorics*, DS6.
[3] Hartsfield, N., & Ringel, G. (1990). *Pearls in graph theory*. Academic Press.
[4] Baca, M., & Miller, M. (2008). *Super edge antimagic graphs – A wealth of problems and some solutions*. Brown Walker Press.
[5] Macdougall, J. A., Miller, M., Slamin, & Wallis, W. D. (2002). Vertex magic total labelling of graphs. *Utilitas Mathematica, 61*, 3–21.
[6] Macdougall, J. A., Miller, M., & Wallis, W. D. (2002). Vertex magic total labelling of wheels and related graphs. *Utilitas Mathematica, 62*, 175–181.
[7] Packiavathi, P. G. P., Balamurugan, S., & Gnanajothi, R. B. (2020). Hop domination number of caterpillar graphs. *Advances in Mathematics: Scientific Journal, 9*(5), 2739–2748.
[8] Sedlacek, J. (1976). On magic graphs. *Mathematica Slovaca, 26*, 329–335.
[9] Sreenivasan, R. (2026). Vertex antimagic edge slither labeling of caterpillar graphs. *Journal of Electronics and Information Technology, 26*(3), 28–30.