



Vertex Antimagic Edge Slither Labeling of Grid Graphs

Abstract: The grids are one of the most intriguing concepts that have wider area of applications in the field of science and technology. They are easy to construct and comprehend. The grids play a very important role specially in the networks and circuits. In this paper, the admissibility of vertex antimagic edge slither labeling, a special case of antimagic labeling to the grid graph $G(1, n)$ is proved.

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1.Introduction

Malaria is a life-threatening disease caused by The concept of labeling has come a far way since its origin. Many researchers have introduced a variety of labeling and the one that stands out is the conception of vertex antimagic edge labeling. This concept has already been applied to grid graphs, and the grid graphs have been proved to accept this notion. This paper deals with a manipulation of the antimagic labeling called as antimagic slither labeling.

2. Vertex Antimagic Edge Slithering Labeling

In graph theory, the idea of weight of a vertex or an edge refers to the assignment of positive integers to the vertices or edges or both. The definition of antimagic labeling to the graphs was first given by Baca and Miller [4], and they defined a graph to be an (a, d) antimagic edge labelled graph if there exists a positive integer a and a non-negative integer d and a bijection $g: E \rightarrow \{1, 2, \dots, |E(G)|\}$ such that the induced mapping $g^*: V(G) \rightarrow W$, where $W = \{a, a+d, a+2d, \dots, a+(|V|-1)d\}$ is also a bijection. The introduction to the concept of magic labeling was first done by J. Sedlacek [8]. Then it was J.A. Macdougall, M. Miller and W.D. Wallis [5] [6] who extended the notion of magic labeling to vertex magic labeling. N. Hartsfield and J. Ringel [3] were the first to introduce antimagic labeling. Later it was R. Bodendiek and G. Walther [1] who gave the concept of (a, d) antimagic labeling. J.A. Gallian [2] compiled all the types of labeling to different types of graphs. The concept of vertex antimagic edge slither labeling was a manipulation of vertex antimagic edge labeling wherein the labeling to the edges of the graph is done based on the slither movement of snakes.

Grid Graphs: Grid graphs are special type of tessellation of C_4 . The tessellation of graphs are studies extensively by Richard Xu and Sergiy Merenkov [7]. Grid graphs are graphs that are undirected graphs where the vertices resemble to a 2D or 3D integer lattice, with edges linking vertices. Importantly, these graphs are the subgraphs of an infinite grid, and these graphs are classical definition of planar graphs.

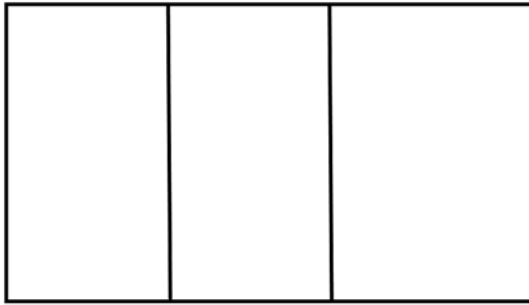
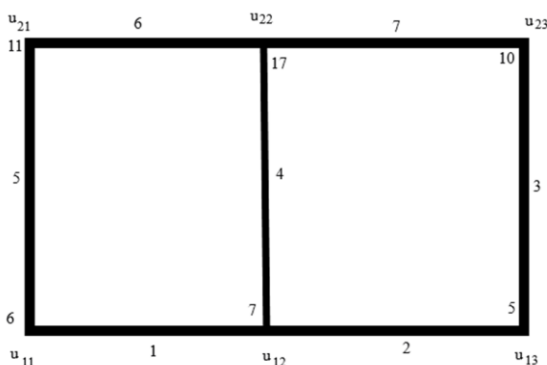


Figure.1 G(1, 3) grid graph

Main Result: The main result of this paper is to prove the applicability of vertex antimagic edge slither labeling to the grid graph $G(1, n)$.

Theorem: The grid graph $G(1, n)$, $n \geq 2$, admits vertex antimagic edge slither labeling.

Proof: Consider the grid graph given by $G(1, n)$, $n \geq 2$. Let $n = 2$. The graph $G(1, 2)$ is given as follows



To prove that the edge labels of the graph are distinct, the labels of the edges are defined by the function $g: E \rightarrow \mathbb{R}$ as follows

$$g(u_{ij} u_{i(j+1)}) = j, i = 1, 1 \leq j \leq 3$$

$$g(u_{ij} u_{i(j+1)}) = i + j + 2n - 1, i = 2, 1 \leq j \leq 3$$

$$g(u_{(i+1)j} u_{ij}) = 2(n + i) - j, i = 1, 1 \leq j \leq 3$$

Claim: The edge labels are all distinct.

Case – 1: For $j \neq k$, $i = 1$, $1 \leq j \leq 3$, assume $g(u_{ij} u_{i(j+1)}) = g(u_{ik} u_{i(k+1)})$

$$\Rightarrow j = k, \text{ a contradiction.}$$

$$\Rightarrow \text{the edge labels are distinct.}$$

Case – 2: For $j \neq k$, $i = 2$, $1 \leq j \leq 3$, assume $g(u_{ij} u_{i(j+1)}) = g(u_{ik} u_{i(k+1)})$

$$\Rightarrow j = k, \text{ a contradiction.}$$

$\Rightarrow$ the edge labels are distinct.

Case – 3: For $j \neq k$, $i = 1$, $1 \leq j \leq 3$, assume $g(u_{(i+1)j} u_{ij}) = g(u_{(i+1)k} u_{ik})$

$$\Rightarrow j = k, \text{ a contradiction.}$$

$$\Rightarrow \text{the edge labels are distinct.}$$

Case – 4: For $i_1 = 1$ and $i_2 = 2$, $1 \leq j \leq 3$, assume $g(u_{i_1 j_1} u_{i_1(j_1+1)}) = g(u_{i_2 j_2} u_{i_2(j_2+1)})$

$$\Rightarrow j_1 = i_2 + j_2 + 2n - 1$$

$$\Rightarrow j_1 = 2 + j_2 + 2n - 1$$

$$\Rightarrow j_1 = j_2 + 2n + 1, \text{ a contradiction.}$$

$$\Rightarrow \text{the edge labels are distinct}$$

Case – 5: For $i_1 = 1$ and $i_2 = 2$, $1 \leq j \leq 3$, assume $g(u_{i_1 j} u_{i_1(j+1)}) = g(u_{i_2(j+1)} u_{i_2 j})$

$$\Rightarrow j = 2(n + i_2) - j$$

$$\Rightarrow 2j = 2(n + 2)$$

$$\Rightarrow j = (n + 2), \text{ a contradiction}$$

$$\Rightarrow \text{the edge labels are distinct}$$

Case – 6: For $i_1 = 2$ and $i_2 = 1$, $1 \leq j \leq 3$, assume $g(u_{i_1 j} u_{i_1(j+1)}) = g(u_{i_2(j+1)} u_{i_2 j})$

$$\Rightarrow i_1 + j + 2n - 1 = 2(n + i_2) - j$$

$$\Rightarrow 1 + j + 2n - 1 = 2(n + 2) - j$$

$$\Rightarrow (n + j) = (n + 2),$$

$$\Rightarrow j = 2, \text{ a contradiction}$$

$$\Rightarrow \text{the edge labels are distinct}$$

From the above cases, it is clear that the edge labels are all distinct. The label of a vertex is the sum of the labels of edges that are incident with the vertex and since all the edge labels are distinct, the vertex labels are all distinct as well. Hence the grid graph $G(1, n)$, $n \geq 2$ admits vertex antimagic edge slither labeling.



3. Conclusion

In this paper, a manipulation of the vertex antimagic edge labeling namely vertex antimagic edge slither labeling is applied to a special type of tessellation, the grid graphs and it has been proved that the graph admits vertex antimagic edge slither labeling. The same labeling concept can be applied to the graph $G(m, n)$, $m > 1$, $n \geq 3$ and can be verified for the admittance of vertex antimagic edge slither labeling.

Conflict of Interest. There is none among the authors

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